

LOCAL LINEARIZATION METHODS IN SPACE AND TIME FOR NONLINEAR WAVE PROPAGATION

Juan I. Ramos
Professor
Office 2-139-D
E.T.S. Ingeniería Industrial
Universidad de Málaga
Dr. Ortiz Ramos, s/n
29071 Málaga
Spain

Tel: +34-951952387
E-mail: jirs@lcc.uma.es

Abstract. In this keynote lecture, linearization methods for nonlinear ordinary (odes) and partial differential equations (pdes) are presented. For initial-value problems of odes, these linearization methods provide locally exponential solutions, whereas, for boundary-value problems of odes, local spatial linearization subject to continuity of the dependent variables and their first derivative provides locally analytical solutions. For one-dimensional transport equations, upon first discretizing the time derivative and then linearizing the resulting equations with respect to the previous time level, two-point nonlinear boundary-value problems are obtained. These problems become locally linear and integrable upon local linearization in space. For multidimensional problems, time discretization followed by time linearization and approximate factorization of the resulting multidimensional nonlinear elliptic problem into a sequence of one-dimensional ones yields nonlinear two-point boundary-value problems which upon local spatial linearization result in linear ones that have local analytical solutions. The advantages and disadvantages of time linearization are discussed in detail, and the methods are applied to solve initial-value problems of nonlinear dynamical systems including nonlinearities that are not differentiable, differential-difference equations, delay equations, jerk dynamics, nonlinear two-point boundary-value problems with regular singular points at a boundary, one- and two-dimensional advection-reaction diffusion problems arising in combustion theory, one-dimensional nonlinear acoustic phenomena, and the break-up and formation of solitary waves from both the equal-width and the generalized regularized-long wave equations. Comparisons between the results obtained with these methods and standard second-order accurate finite-difference discretization and three-point, compact, fourth-order accurate discretization of the spatial derivatives are also presented.

Keywords: time linearization, local spatial discretization, nonlinear evolution equations, nonlinear wave propagation, reaction-diffusion equations, nonlinear dynamics