Analogies between central finite difference, Hencky bar chain and Eringen's nonlocal beam models for buckling and vibration

***Abstract*—**This keynote lecture points out the analogies between the central finite difference beam model, Hencky bar-chain model and Eringen's nonlocal beam model. The finite difference beam model is obtained by discretizing the differential governing equation of the beam via finite difference equations for approximating the derivatives. The Hencky bar-chain model comprises finite rigid beam segments connected by frictionless hinges with elastic rotational springs. Eringen's nonlocal theory allows for the effect of small length scale effect which becomes significant when dealing with nanobeams. Based on the mathematically similarity of the governing equations of these three beam models, analogies exist between them. The consequence is that one could readily obtain the buckling and vibration solutions of beams without solving the differential equation as well as it allows one to calibrate Eringen's small length scale coefficient e_0. As an example, for an initially stressed vibrating beam with simply supported ends, it is found via this analogy that Eringen's small length scale coefficient $e_0 = \sqrt{\frac{1}{6} - \frac{1}{12}\frac{\sigma_0}{\breve{\sigma}_m}}$ where σ_0 is the initial stress and $\breve{\sigma}_m$ is the m-th mode buckling stress of the corresponding local Euler beam. It is shown that e_0 varies with respect to the initial axial stress, from $1/\sqrt{12}$ at the buckling compressive stress to $1/\sqrt{6}$ when the axial stress is zero and it monotonically increases with increasing initial tensile stress. The small length scale coefficient e_0, however, does not depend on the vibration/buckling mode considered.

Keywords: buckling, finite difference beam model, Hencky bar chain model, nonlocal beam theory, repetitive cells, small length scale coefficient, vibration